

B2ASP

A Clingo SAT backend for ProB



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Scryer Prolog Meetup

STUPS Group Core Expertise

Software Engineering and Programming Languages Group

- **Formal Methods:**

- ProB validation toolset: animation/simulation, constraint solving, test-case generation, model checking, data validation, EN51028 T2 certified, visualisation, validation obligation manager
- Modelling & verification of safety critical systems (hybrid train detection, moving block train control, ...)
- Certification of AI, combining formal models and AI, KI-LOK project
- B, Event-B, TLA+, Alloy, CSP, Z, ...

- **Programming Languages**

- symbolic AI, constraint logic programming
- tool validation, static program analysis, just-in-time compilation

Before starting: Other Work

of potential interest to Stryer Prolog audience

- Partial Evaluation and Program Analysis for Prolog
 - Ecce
- Jupyter Prolog Notebooks for SWI+SICStus; adaptable for Stryer?
- Testing and certification: Prolog fuzzer, Mathematical Laws checking via constraint solving/model checking
- ProB can also animate Prolog rules
 - Inference rules of Event-B, natural deduction visualisation (cf Mark Thom's first talk), see later
- Teaching: Introduction to Logic Programming (A*, Minimax, MCTS, ... in Prolog), Advanced Topics in Logic Programming (writing interpreters, parsers, semantic analysis in Prolog, big-step / small-step semantics, ...)

✓ Well-Formed Formulas (WFF)

All atomic propositions are WFF) If a und b are WFF then so are:

- $(\neg a)$
- $(a \wedge b)$
- $(a \vee b)$
- $(a \rightarrow b)$
- $(a \iff b)$ No other formulas are WFF.

Note in the slides we use the single arrow $\rightarrow$ instead of the double arrow $\Rightarrow$ for implication which is maybe more standard.

Comment: a, b are metavariables outside the syntax of propositional logic.

Maybe this reminds you of formal grammars from a theoretical computer science lecture. And indeed, the above can be written as a grammar using a non-terminal symbol wff.

In Prolog, grammars can actually be written using DCG notation, which we will see and understand much later in the course. Here we simply write the grammar in Prolog style and can then use it to check if a formula is a WFF:

```
| :- set_prolog_flag(double_quotes, codes).  
wff --> "p". % atomic proposition  
wff --> "q". % atomic proposition  
wff --> "(\neg,wff,")".  
wff --> "(", wff, "\wedge", wff, ")".  
wff --> "(", wff, "\vee", wff, ")".  
wff --> "(", wff, "\rightarrow", wff, ")".  
wff --> "(", wff, "\iff", wff, ")".
```



ECCE

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```
q(c,d).
q(d,e).

reduce_add(List,Res) :-
    reduce(add,0,List,Res).
add(X,Y,Z) :-
    Z is X + Y.

rev(L,R) :-
    rev(L,[],R).

rev([],L,L).
rev([H|T],A,R) :-
    rev(T,[H|A],R).
```

GOAL

ACTIONS

[Specialise](#)[Slice](#)[MSV](#)[RAF](#)[FAR](#)

SPECIALISED CODE

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```
/* Specialised Predicates:
test__1(A) :- test(A).
q__3(A,B) :- q(A,B).
map__2(A,B) :- map(q,A,B).
*/

test(A) :-
    test__1(A).
test__1([],[]).
test__1([A|B],[C|D]) :-
    q__3(A,C),
    map__2(B,D).
map__2([],[]).
map__2([A|B],[C|D]) :-
```

WORKING EXAMPLES

vanilla_list.pl

[Load](#)

CONTROL SETTING

- ☒ Default Conjunctive
- ☐ Conjunctive Fast
- ☐ Classic
- ☐ Mixtus Style
- ☐ Classic Fast
- ☐ Minimal
- ☐ Termination

POST-PROCESSING

- ☐ Max
- ☒ Default
- ☐ Off

PLUG-INS

[RUL](#)[RUL-BUP](#)[CONVEX-LL](#)[CONVEX-TS](#)[About Plug-Ins](#)

```
>>> :print @FUZZ
@FUZZ = #id1.(id1 : POW(String) & {id0|id0 : REAL & ({ } = id1\ / id1 & id1 / \ id1 = { })} : FIN({id0|id0 : REAL & ({ }
= id1\ / id1 & id1 / \ id1 = { })}))
∃id1.(
  {id0|(∅ = id1\ / id1 ∧ id1 / \ id1 = { })} ∈ FIN({id0|(∅ = id1\ / id1 ∧ id1 / \ id1 = { })})
)

>>> :print @FUZZ
@FUZZ = #id0.(id0 : POW(INTEGER * POW(INTEGER * POW(String))) & last(last(id0)) /<<: inter({RANGE_LAMBDA__|
RANGE_LAMBDA__ : POW(String) & RANGE_LAMBDA__ = String}))
∃id0.(
  last(last(id0)) ∉ inter({ρ|ρ = String})
)

>>> :print @FUZZ-ARITH
@FUZZ = btrue
T

>>> :print @FUZZ-ARITH
@FUZZ = id0 : POW(INTEGER * (INTEGER * INTEGER) * INTEGER) & id1 : POW(INTEGER * (INTEGER * INTEGER) * INTEGER) &
(MAXINT /: { } & MAXINT > 75 => prj1(NATURAL1,pred) /<<: id0 \ / id1)
(
  (
    MAXINT ∉ ∅
    ∧
    MAXINT > 75
  )
  =>
  prj1(N1,pred) ∉ id0 ∪ id1
)
```

MACHINE ArithmeticLaws

DEFINITIONS

MAX_x == 50;

MAX_y == 9;

MAX_z == 4;

MIN_x == -3;

MIN_y == -3;

MIN_z == -3

VARIABLES x, y, z

INVARIANT

x:INTEGER & y:INTEGER & z:INTEGER &

x*y = y*x &

x*(y+z) = x*y + x*z &

x+y = y+x &

x*1 = x &

1*x = x &

x*0 = 0 &

0*x = 0 &

(x+y)+z = x + (y+z) &

(x*y)*z = x*(y*z) &

2*x = x+x &

x**2 = x*x &

(x>=0 => 1**x = 1) &

(x>=0 => ((x / 2)*2 = x) <=> (x mod 2 = 0))) &

(x>0 => 2**x = 2*(2**(x-1))) &

(x>0 => 2**(10*x) = 2*(2**(10*x-1))) &

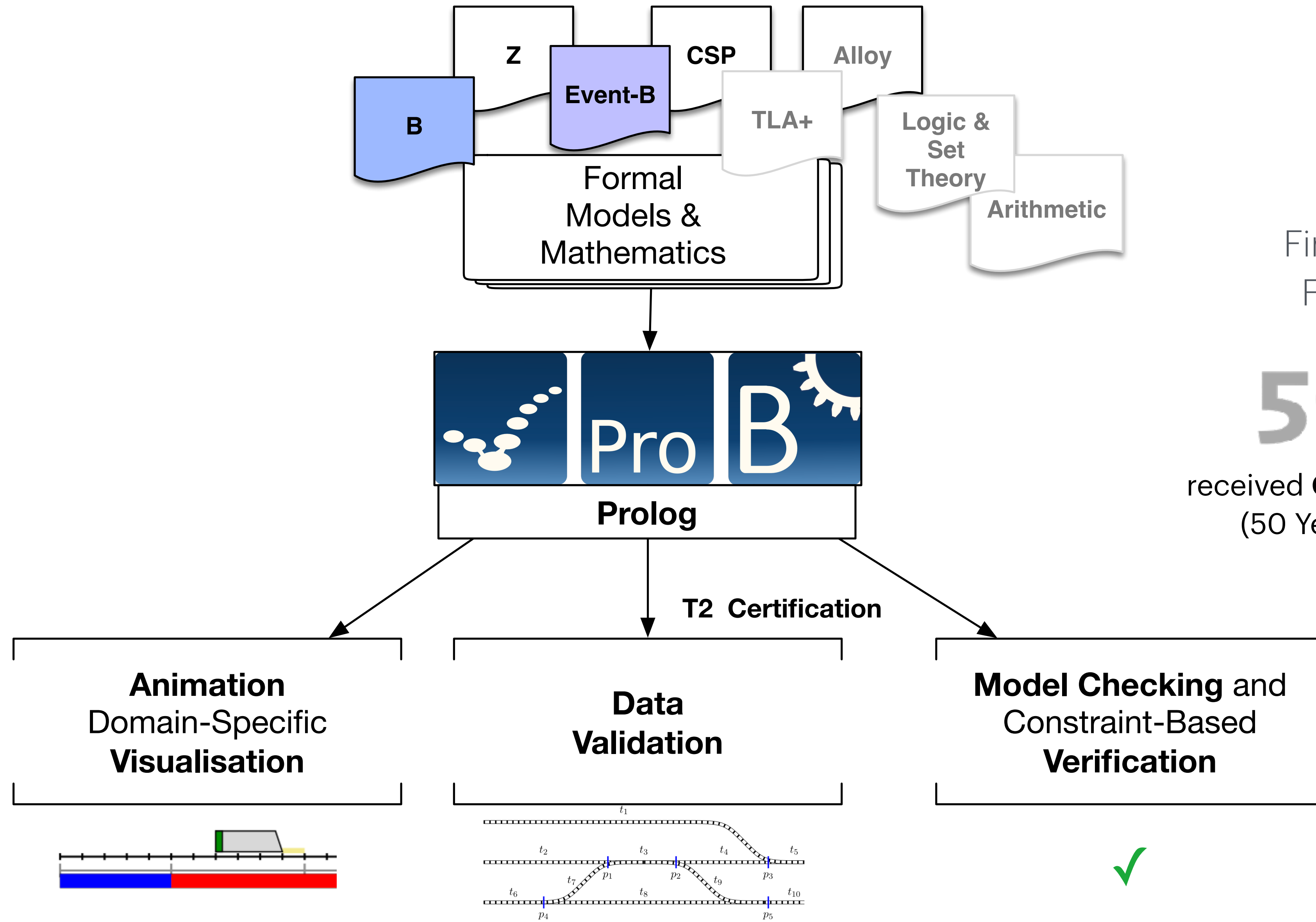
(x>0 => 3**(10*x) = 3*(3**(10*x-1))) &

(x>y or x<=y) &

(x>y or x=y or x<y) &

&...

Validation Tool for Formal Models



First article
FM'2003

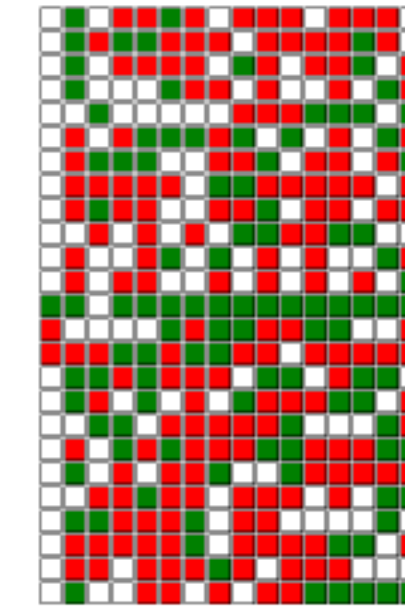
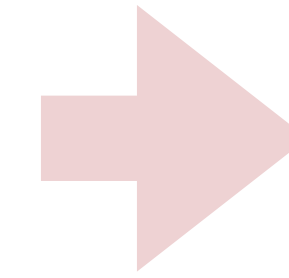
50 | 1972
2022

received **Colmerauer Prize**
(50 Years of Prolog)

B2ASP

Essentials

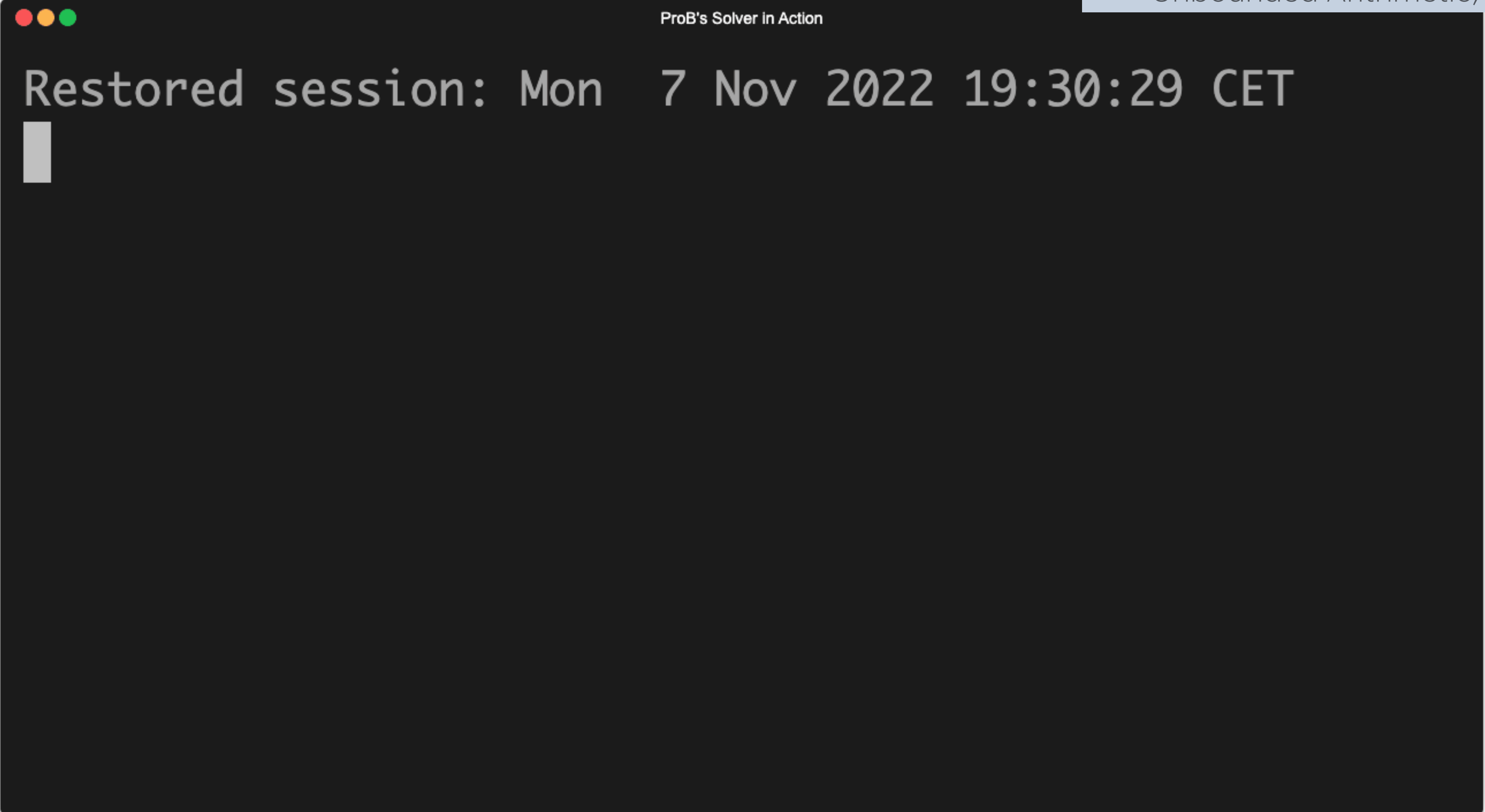
$\{B\}$



- New translation of **B** (Predicate Logic, Higher-Order Set Theory, Arithmetic) to **SAT** and back
- Available in **ProB** and applicable to other state-based formalisms (**TLA+**, **Z**, ...)
- Technique fully rooted in Prolog and Logic Programming: CLP(FD) Bounds Analysis, Prolog Translation from B to **ASP** (Answer Set Programming) and back (probably could be run in Stryer Prolog)
- Not a general purpose solver, but very useful for dedicated **constraint satisfaction** and **optimisation** problems and symbolic verification

ProB's Solver in Action

Demo: Set Unification, Set Comprehension with open variable, Unicode, Sets of Sets,, Unbounded Arithmetic, ...

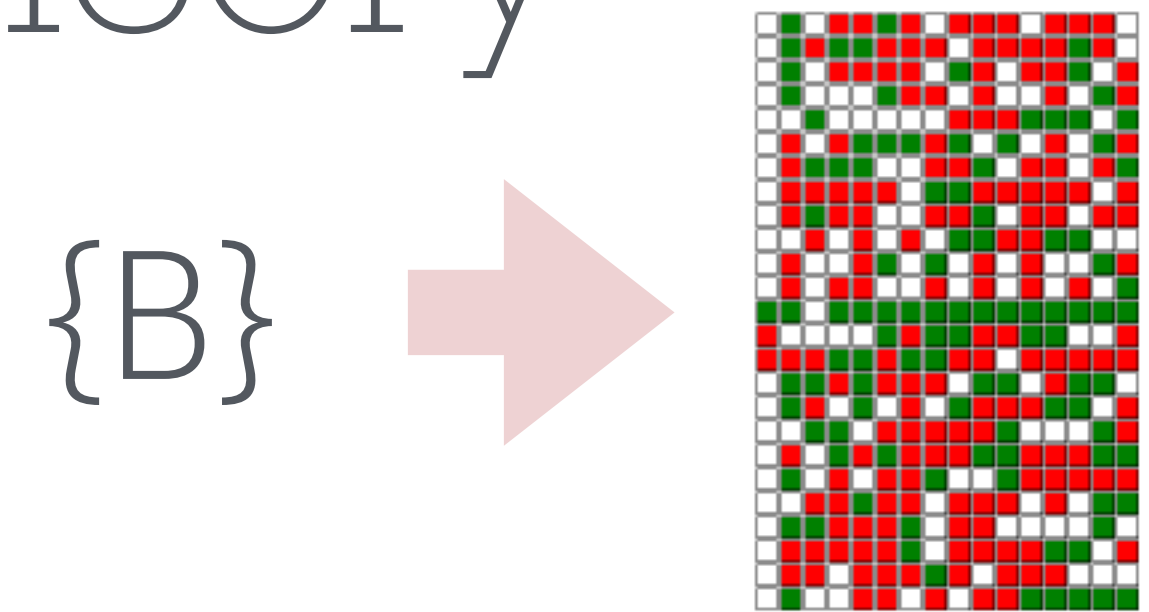


```
ProB's Solver in Action

Restored session: Mon  7 Nov 2022 19:30:29 CET
█
```


Alternatives to Solving B / Set Theory

ProB default: CLP(FD) + co-routines + attributed vars



- **CHR** (Constraint Handling Rules)
 - already used (optionally) for a few constraints
 - tricky to make a robust & fast solver
- **SMT** (Z3, CVC4) [SchmidtLeuschel: Int. J. Softw. Tools Technol. Transf. 24(6): 1043-1077 (2022)]
 - ok for unsatisfiable constraints,
 - not yet good for model finding

ASP (Answer Set Programming)

Basics

- Normal logic program (i.e., with negation)
- Compute stable models: typically by “grounding” and SAT solving
- **Stable** model of a logic program (called answer set): is a model in classical logic, where all atoms in the model are justified by some rule
- Example: three classical logic models: $\{q,p\}$, $\{q,r\}$, $\{p,q,r\}$. Only $\{q,p\}$ is stable.

```
q.  
p :- q, not r
```

ASP (Answer Set Programming)

Language Constructs

- Horn clauses, with variables (aka Datalog)
- Disjunctions in head
- Integrity Constraints
- Choice
- Aggregates

```
mortal(X) :- human(X).
```

```
p(X) ; q(X) :- r(X).
```

```
:- edge(X,Y), col(X,C), col(Y,C).
```

```
1 { p(X) } 2.
```

```
... #sum{.....}
```